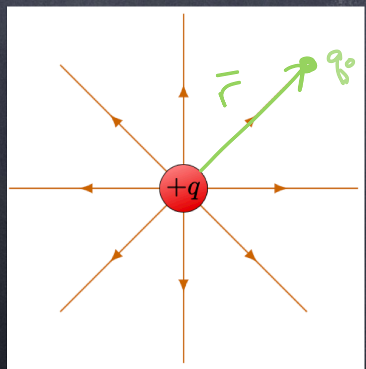
PHY 117 HS2023

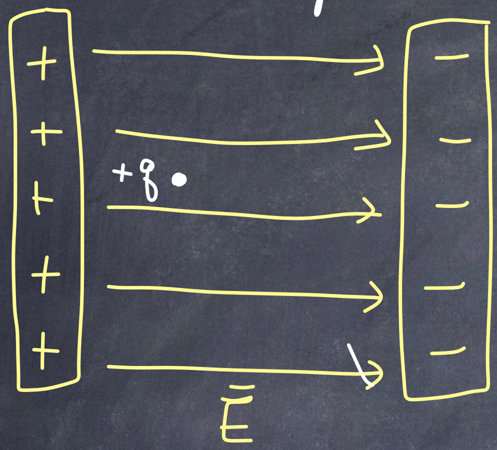
Week 8, Lecture 2
Nov. 8th, 2023
Prof. Ben Kilminster

yesterday :



$$\bar{E} = \frac{kq}{r^2}\,\hat{r}$$

What will happen to a charge $+q$ in between 2 planes of + and − charge?



We know that $\bar{F} = q\bar{E}$ and $\Sigma \bar{F} = m\bar{a}$

$$\Sigma \bar{F} = m\bar{a}$$

$$q\bar{E} = m\bar{a}$$

so $$\bar{a} = \frac{q\bar{E}}{m}$$

This is the same direction as $\bar{E}$ because it is a (+) charge.

The charge will accelerate!

The $\bar{E}$-field points the way a (+) charge moves.

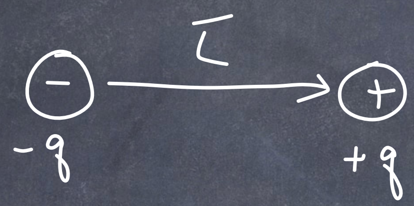
For an electron, $q = -e$, so $\bar{a} = \frac{-e\bar{E}}{m}$

(opposite $\bar{E}$-field)

If velocity $> 0.1c$ (c = speed of light $= 3E8 \frac{m}{s}$), then you need relativistic equations.

Electric dipole:

A system of two equal & opposite electric charges separated by a small distance.



$\bar{p}$ = electric dipole moment $= q\,\bar{l}$

$\bar{l}$ = distance vector

What is the E-field of a dipole?

$$E = \frac{+kq}{(x-a)^2}\hat{x} + \frac{-kq}{(x+a)^2}\hat{x}$$

$\bar{E}$ of $+q$ $\qquad$ $\bar{E}$ of $-q$

simplify:

$$\bar{E} = kq\hat{x}\left(\frac{(x+a)^2-(x-a)^2}{(x-a)^2(x+a)^2}\right)$$

$$\bar{E} = kq\hat{x}\left(\frac{\cancel{x^2} + 2xa + \cancel{a^2} - \cancel{x^2} + 2xa - \cancel{a^2}}{((x-a)(x+a))^2}\right)$$

$$\bar{E} = \frac{kq\hat{x}(4xa)}{(x^2-a^2)^2}$$ substitute in $p = q\bar{L} = 2aq\hat{x}$

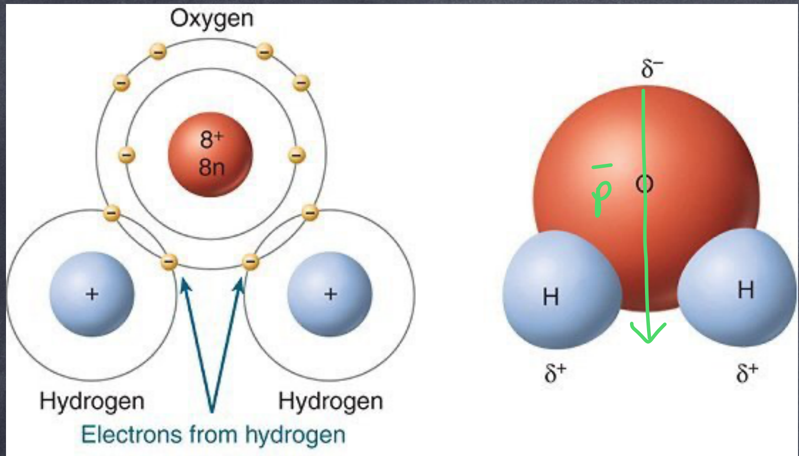
$$\bar{E} = \frac{k\bar{p}\,2x}{(x^2-a^2)^2} = \frac{2k\bar{p}x}{x^4 - \underbrace{2x^2a^2 + a^4}}$$

Approximate if $x \gg a$, then this part is small

$$\bar{E} \cong \frac{2k\bar{p}\cancel{x}}{x^{\cancel{4}3}} = \frac{2k\bar{p}}{x^3} = \frac{2kLq\hat{x}}{x^3}$$

$\boxed{\text{Electric field of a dipole for distances } x \gg L, \text{ along } x\text{-axis is } \bar{E} = \frac{2k}{x^3}\bar{p} \text{ where } \bar{p} = \bar{L}q}$

practical dipole: H_2O molecule is a permanent dipole, a "polar" molecule.



(a) Electron shells in a water molecule

(b) Distribution of partial charges in a water molecule

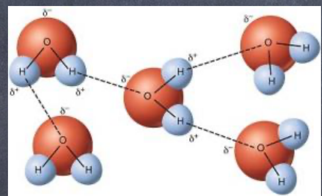
$L \sim 0.1\,\text{nm} = 0.1\,E{-9}\,\text{m}$

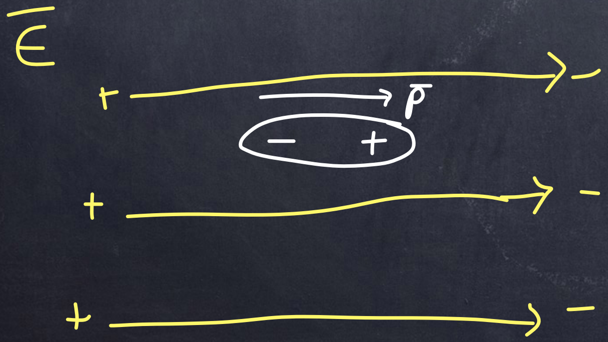
The effective charges depend on how tightly the electrons are bound. (Not obvious, $q \sim 0.8e$)

$\bar{p} = q\bar{L} = (0.8e)(0.1\,\text{nm})$

$\bar{p} = 0.08\,e \cdot \text{nm}$

Electrons are held closer to the oxygen



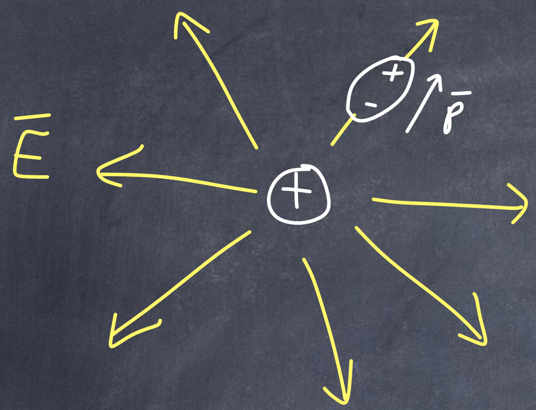


In an E-field, a dipole will rotate and align with $\bar{E}$

Note: $\bar{p}$ vector goes from − to +

$\bar{E}$ vector goes from + to −

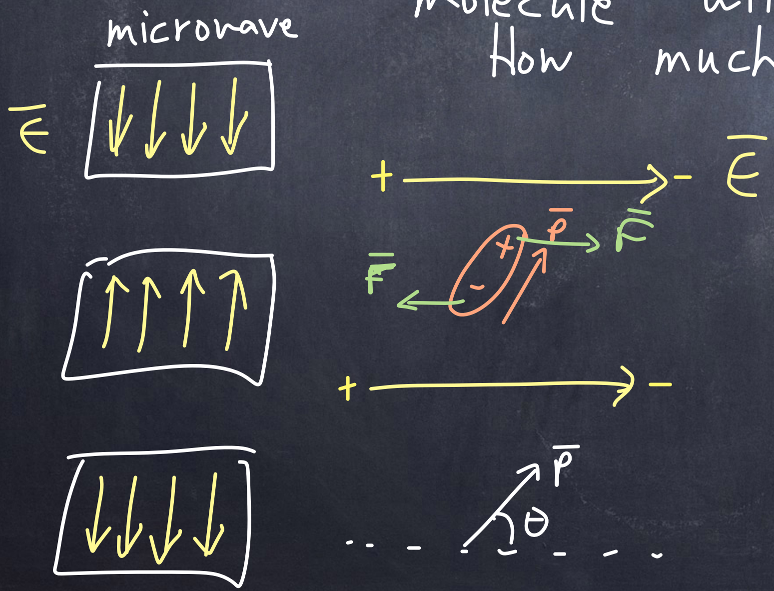
In a non-uniform $\bar{E}$-field, dipole feels a force.



$\bar{E}$-field is stronger closer to the (+) charge.

The dipole feels a force because (−) charge is closer and feels a stronger E-field than the (+) charge.

Example: water in a microwave oven. Electric field changes with an oscillating E-field. water molecule will rotate, generating heat. How much heat are we talking about?



There is a torque from the $\bar{E}$-field that will rotate the dipole.

$$\bar{\tau} = \bar{r} \times \bar{F} \qquad \bar{p} = q\bar{r} \qquad \bar{r} = \frac{\bar{p}}{q}$$

$$\bar{F} = q\bar{E}$$

$$\tau = \bar{r} \times \bar{F} = \frac{\bar{p}}{\cancel{q}} \times \cancel{q}\bar{E} = \bar{p} \times \bar{E} = pE\sin\theta$$

The work to rotate the molecule by $d\theta$

$$\boxed{dW = -\tau\, d\theta} = -pE\sin\theta\, d\theta$$

Electric field is doing the work, so $W = (-)$

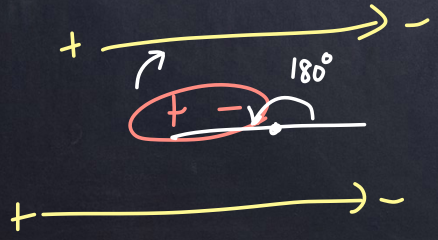
$$W = \int_{\theta_1}^{\theta_2} -\tau\, d\theta = \int_{\theta_1}^{\theta_2} -pE\sin\theta\, d\theta = pE\cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$W = pE(\cos\theta_2 - \cos\theta_1)$$

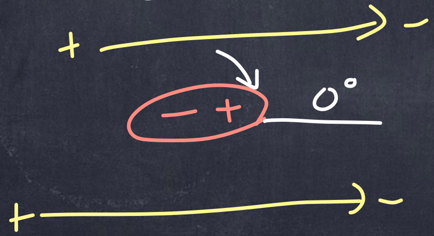
$\theta_1 = 180°$
$\theta_2 = 0°$

To rotate the H_2O molecule by $180°$,

initial
$\theta_1 = 180°$



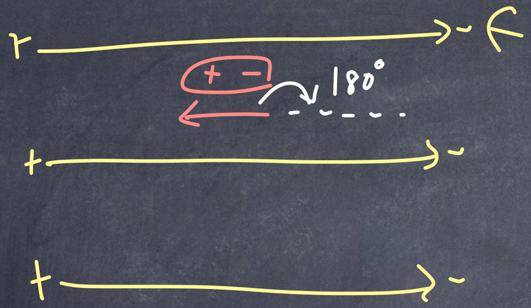
final
$\theta_2 = 0°$



$$W = pE(\cos\theta_2 - \cos\theta_1)$$

$$= pE(+1 - -1)$$

$$W = 2pE$$



$W = 2pE$

Maximum potential energy, $U = -W$

when $\theta = 180^\circ \Rightarrow \cos\theta = -1$

$$U = -W = -pE\cos(180^\circ) = pE$$

Minimum, when $\theta = 0^\circ$, $\cos\theta = 1$

$$U = -W = -pE\cos(0^\circ) = -pE$$

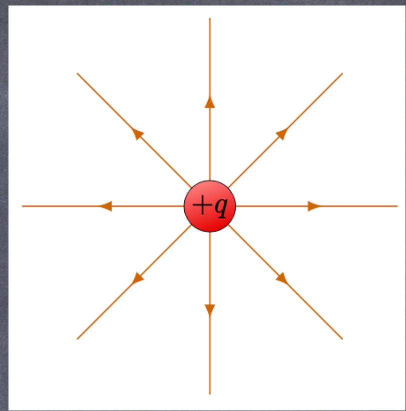
$$\Delta U = -pE - pE = -2pE$$

work is negative of ΔU.

Calculating Electric field using Gauss' Law.

Example: calculate E-field for a charge, $+Q$.

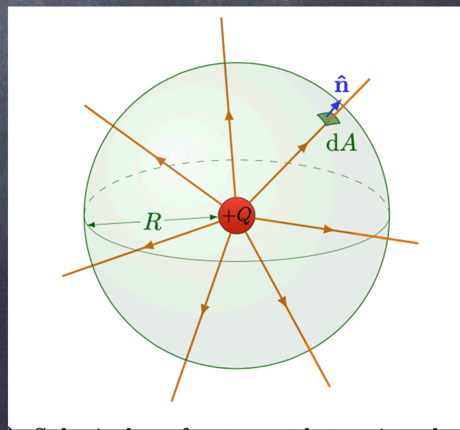
1) First, draw E-field lines.



2) Now draw a closed surface, S, around the charge where $|\bar{E}|$ is constant everywhere or zero.

Example: we draw a spherical shell

3) The closed surface, S, has a normal vector, $\hat{n}$, which is perpendicular to the surface.



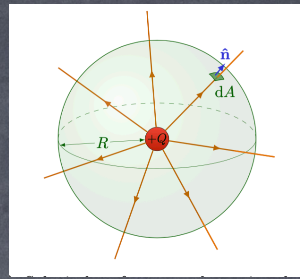
Note: In example, $\bar{E}$ is parallel to $\hat{n}$ everywhere, so

$$\bar{E} \cdot \hat{n} = E$$

$$\uparrow$$

$$|\hat{n}| = 1$$

4) Calculate the area of the closed surface, S. In example: the area of a spherical shell is $A = 4\pi R^2$



we say that $\oint_S dA = 4\pi R^2$

S = surface

$\oint$: means "closed"

$\oint_S dA$ is the sum of all the dA pieces that make up our closed surface, S.

5) Now, we are ready to apply Gauss' Law!

$$\boxed{\oint_S (\bar{E} \cdot \hat{n})\, dA = \frac{Q}{\varepsilon_0}}$$

Gauss' Law

Q : charge inside our surface, S.

In example, $\bar{E} \cdot \hat{n} = E$, and $\oint_S dA = 4\pi R^2$, and E is constant for all R, so

$$\oint_S (\bar{E} \cdot \hat{n})\, dA = \oint_S E\, dA = E \oint_S dA = E(4\pi R^2)$$

6) Finally, we solve Gauss' Law:

$$\oint_S (\bar{E}\cdot\hat{n})\,dA = E(4\pi R^2) = \frac{Q}{\varepsilon_0}$$

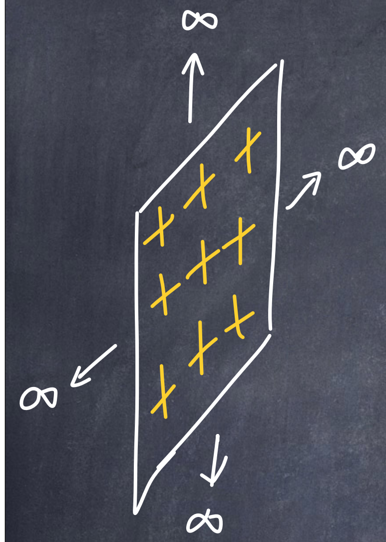
so $E = \dfrac{Q}{4\pi\varepsilon_0 R^2}$ — we add the vector $\bar{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\,\hat{r}$

Done: this agrees with our previous formula for $\bar{E}$-field of a point charge.

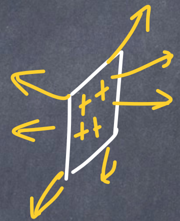
Gauss' Law lets us solve for $\bar{E}$ by finding simple, closed surfaces where $\bar{E}$ is parallel to $\hat{n}$ so that $\bar{E}\cdot\hat{n} = E$, or $\bar{E}$ is perpendicular to $\hat{n}$, $\bar{E}\cdot\hat{n} = E\cdot 1(\cos 90^\circ) = \emptyset$

Note: there are only a few simple cases that can be solved.

What is $\bar{E}$ for an infinite plane of charge, with charge density, $\sigma = \frac{Q}{A}$



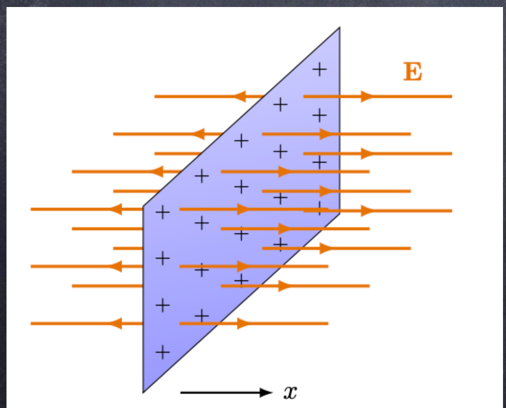
The "infinite" word means we can ignore edge effects



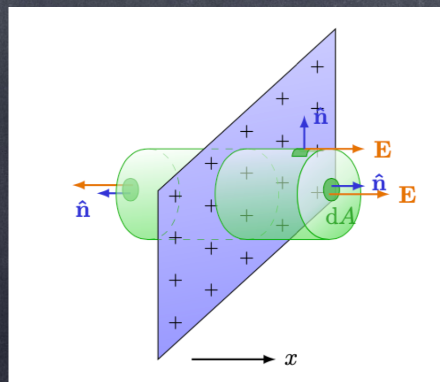
1) we draw E-field lines

2) we draw a closed surface, S, where either $\bar{E} \parallel \hat{n}$ or $\bar{E} \perp \hat{n}$. In example, we choose a "pill box".

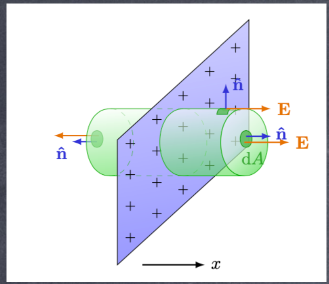
1)

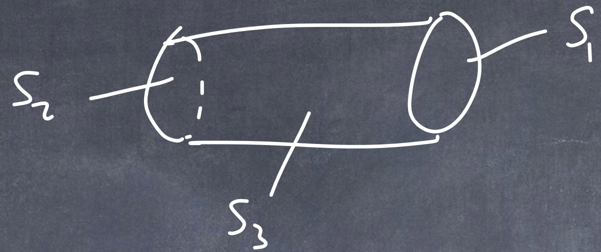


2)



3)





Here: $\bar{E} \parallel \hat{n}$ on $S_1 + S_2$
and $\bar{E} \perp \hat{n}$ on S_3

4) $\oint_S dA$ is the sum of all pieces of the closed surface:

$$\oint_S dA = \underbrace{\int_{S_1} dA}_{A_1} + \underbrace{\int_{S_2} dA}_{A_2} + \underbrace{\int_{S_3} dA}_{A_3}$$

$$= A_1 + A_2 + A_3 \qquad (A_1 = A_2)$$

5) Now, apply Gauss' Law:

on $S_1 + S_2$, $\bar{E} \cdot \hat{n} = E$, and on S_3 $\bar{E} \cdot \hat{n} = 0$

$$\oint_S (\bar{E} \cdot \hat{n})\, dA = \int_{S_1} E\, dA + \int_{S_2} E\, dA + \int_{S_3} 0\, dA = EA_1 + EA_2 + 0$$

$$= 2EA_1$$

How much charge is inside-?

we know that $\sigma = \frac{Q}{A}$, so for any area,

$$Q = \sigma A$$

Here, area $= A_1$, so $Q = \sigma A_1$

Finally, we get:

$$\oint_S (\bar{E} \cdot \hat{n})\, dA = \frac{Q}{\varepsilon_0}$$

$$\downarrow$$

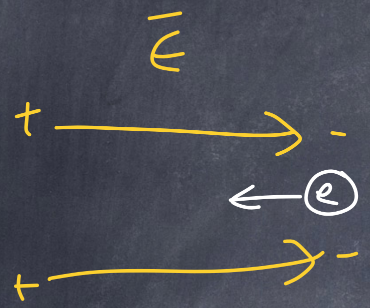
$$2E \cancel{A_1} = \frac{\sigma \cancel{A_1}}{\varepsilon_0}$$

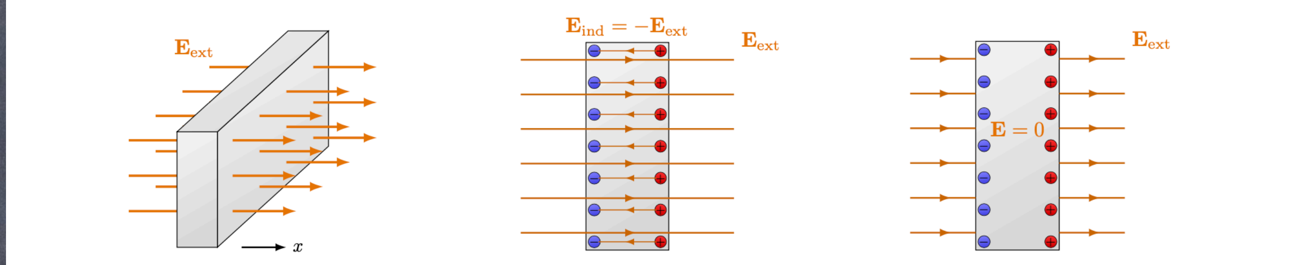
solution: $E = \frac{\sigma}{2\varepsilon_0}$

As a vector: $\bar{E} = \frac{\sigma}{2\varepsilon_0}\hat{x}$ (for $+x$)

$\bar{E} = -\frac{\sigma}{2\varepsilon_0}\hat{x}$ (for $-x$)

If a conductor is in an $\bar{E}$-field, electrons move toward the (+) positive end of $\bar{E}$-field, neutralizing the $\bar{E}$-field inside.





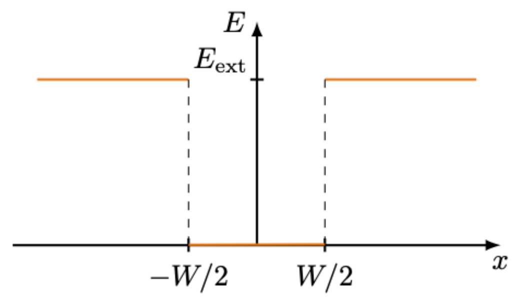
(a) An external field $\mathbf{E}_{\text{ext}}$ is applied to a slab of conducting metal.

(b) The free charges redistribute and create an internal field $\mathbf{E}_{\text{ind}}$.

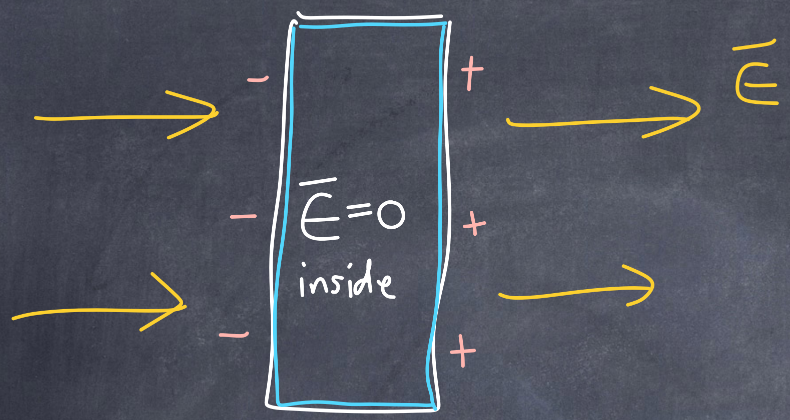
(c) Once the charge has settled, the internal and external fields cancel.

Charge accumulates on the surface of the conductor with ⊖ charges on one side + ⊕ charges on the other side,

Inside conductor, no charges, and $\bar{E} = 0$ inside.



(d) Plot of electric field vs. r.

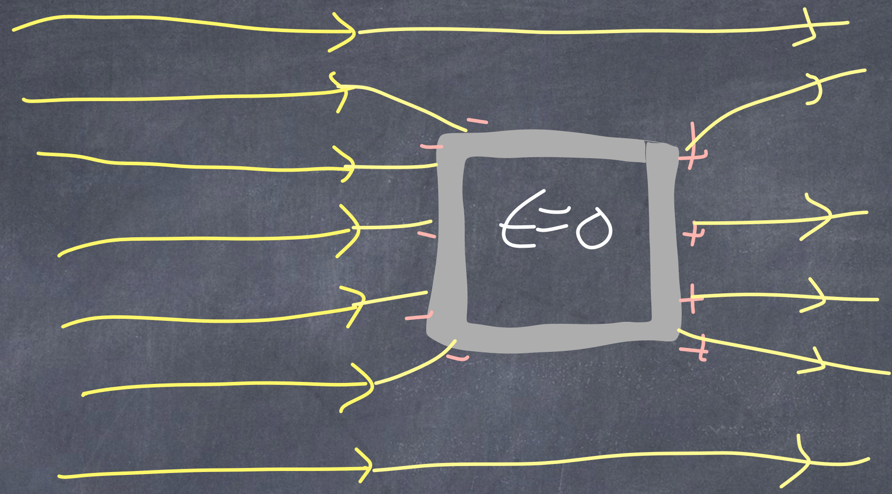


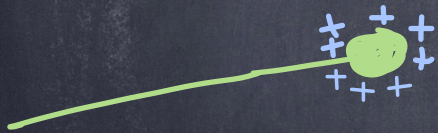
Gaussian surface. $\Rightarrow$ $Q=0$

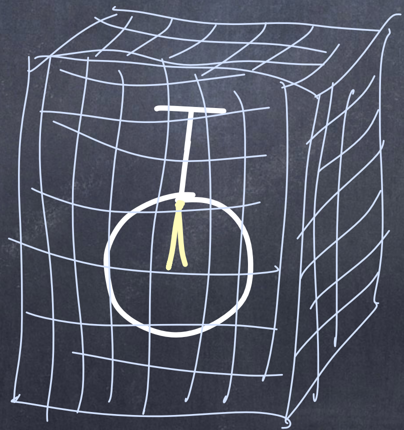
$$\oint \bar{E}\cdot\hat{n}\, dA = \frac{Q}{\varepsilon_0} = 0$$

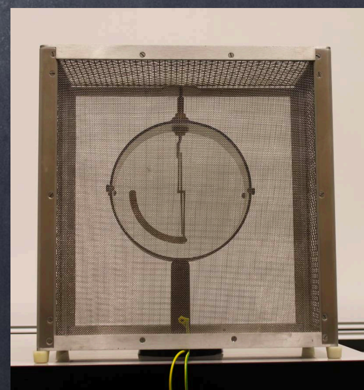
$$\Rightarrow E=0$$

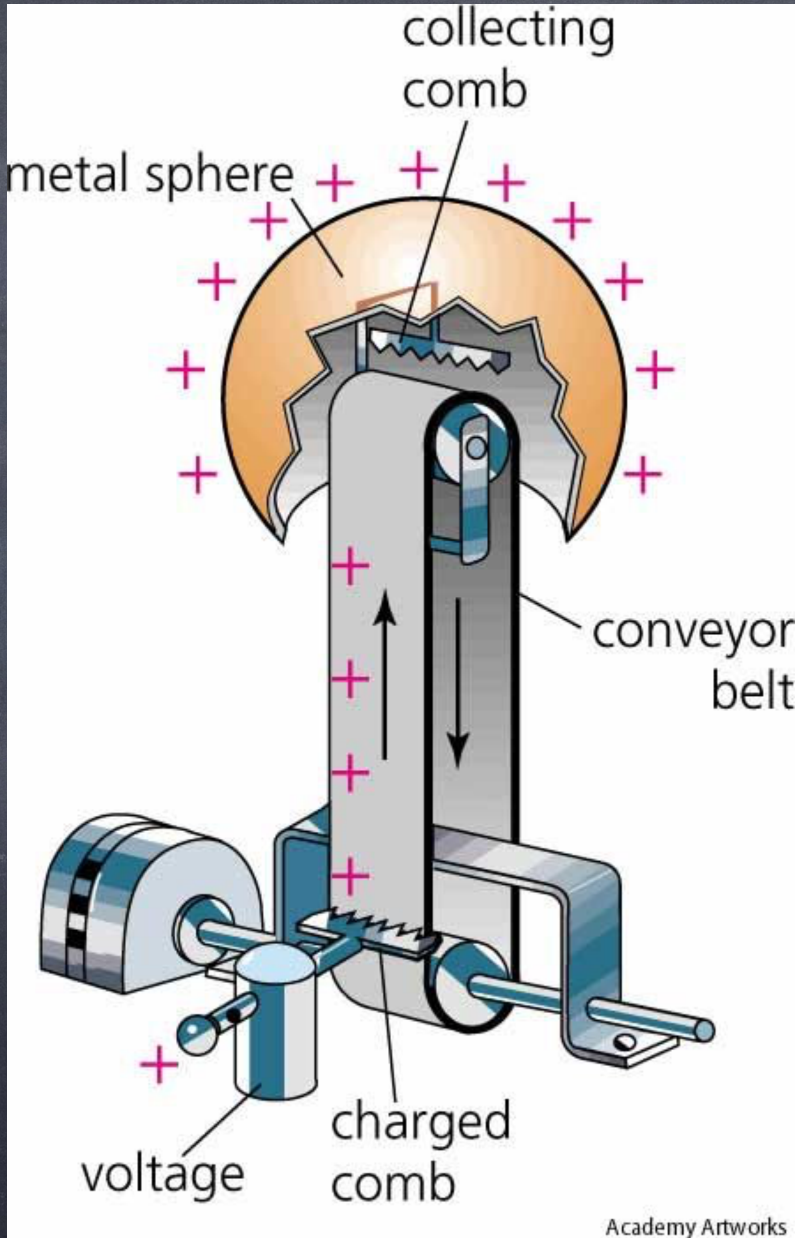
Faraday cage







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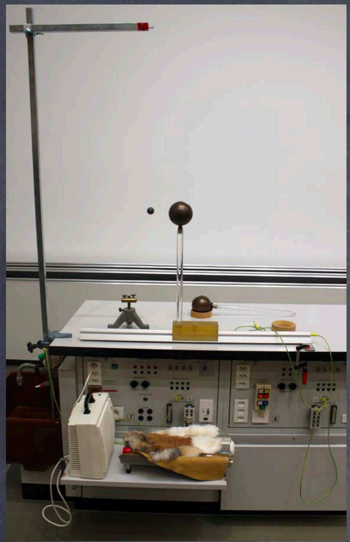

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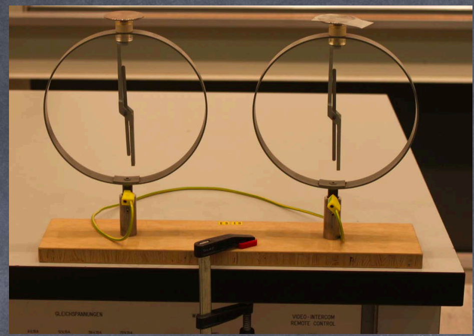
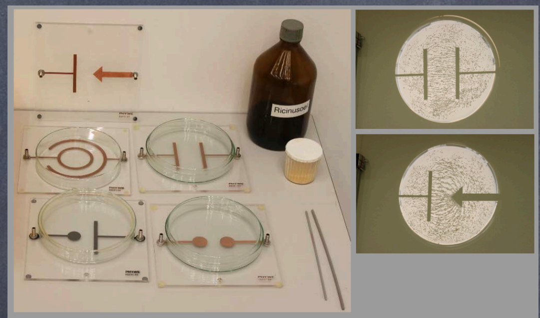
ES8



ES19

ES20



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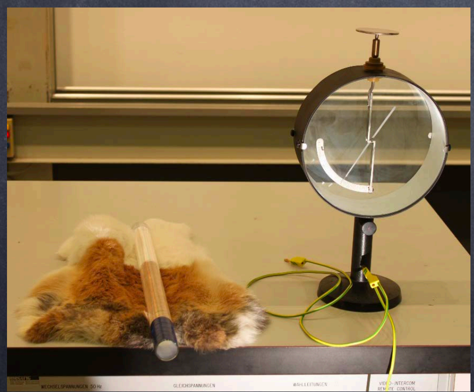

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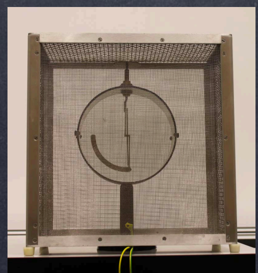

ES26