Let there be light!

PHY117 HS2023

Week 13, Lecture 1
Dec. 12th, 2023
Prof. Ben Kilminster

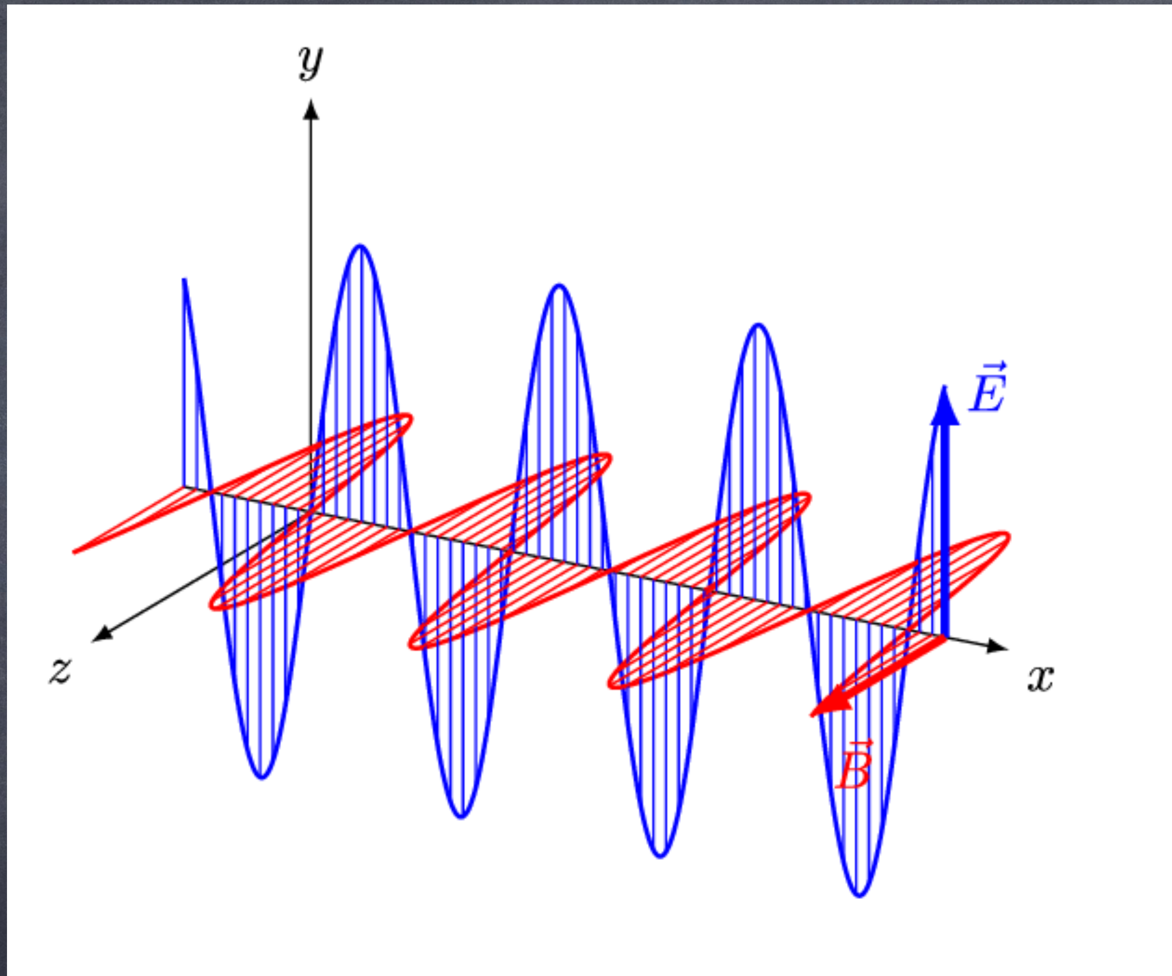
we saw: changing magnetic fields produce electric fields.
changing (moving, for instance) electric fields produce magnetic fields.
This positive feeds generates a self-sustaining phenomenon.

If one solves the four inter-related equations of electricity & magnetism, known as Maxwell's equations (Gauss' Law is one), the solution one finds is a wave, moving at constant speed. The wave is referred to as an electromagnetic wave or "light".

This discovery (Maxwell, 1865) generated other discoveries, like the radio, and paved the way for relativity and our modern understanding of forces and interations.

Electromagnetic (EM waves)

Example of EM wave as a sine wave



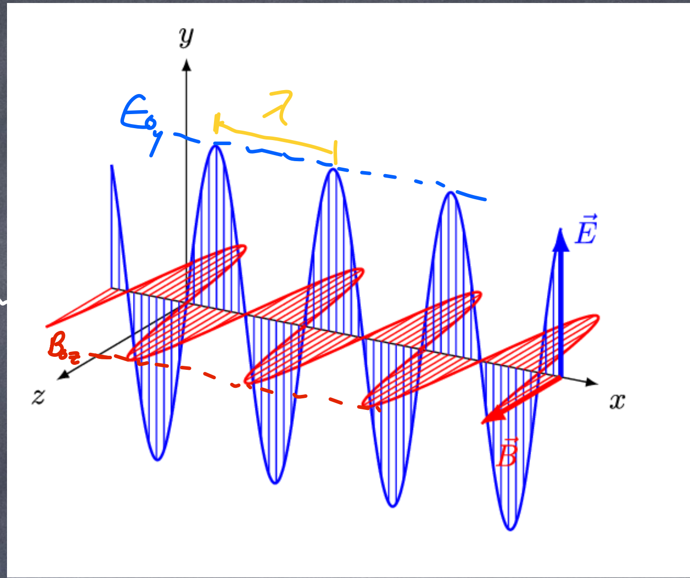
Notice:

1) $\bar{E} \perp \bar{B}$ always
2) $|\bar{E}| \propto |\bar{B}|$ at each moment in time
3) The direction of propagation of our wave ($\bar{V}$) is $\perp$ to $\bar{E}$ & $\bar{B}$

$\longrightarrow \bar{V} \propto \bar{E} \times \bar{B}$ right-hand rule

Using what we know about how to define waves, we can write the wave functions for this:

$\bar{E}$ always in $\pm\hat{y}$ direction
$\bar{B}$ always in $\pm\hat{z}$ direction
propagation always in $+\hat{x}$ direction



EM wave function example ⓪

$$\boxed{\begin{aligned}\bar{E} &= E_{0_y}\sin(kx-\omega t)\,\hat{y}\\ \bar{B} &= B_{0_z}\sin(kx-\omega t)\,\hat{z}\end{aligned}}$$

Since it is a wave, $f\lambda = \frac{\omega}{k}$ = velocity $\equiv c$ = "the speed of light"

we see that $|\vec{E}| \propto |\vec{B}|$. The constant of proportionality is... you guessed it, c.

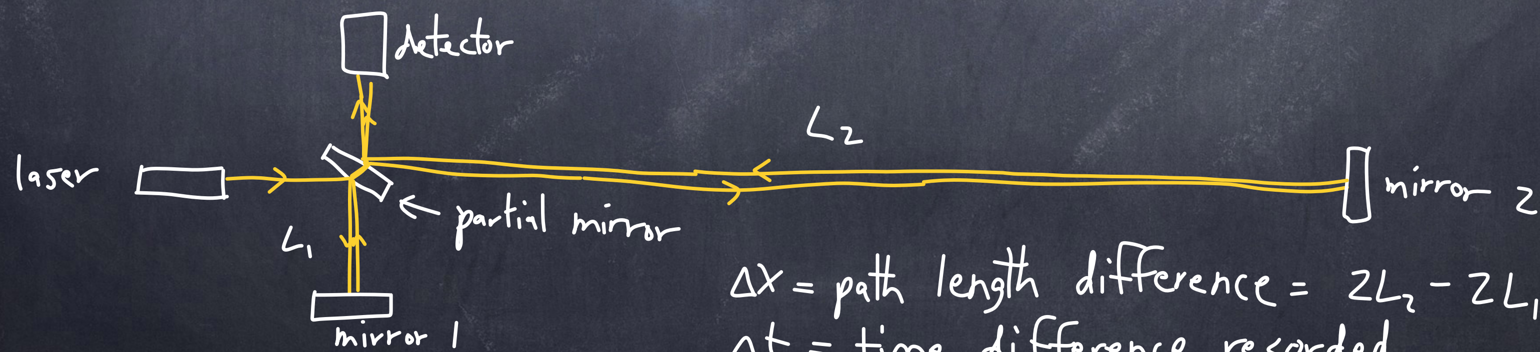
For EM waves, $\boxed{E = cB}$ ① Here $E_y = cB_z$

$$\left[\frac{kg\cdot m}{s^3\cdot A}\right] \quad \left[\frac{m}{s}\right] \quad \left[\frac{kg}{s^2\cdot A}\right]$$

c = exactly $299\,792\,458\ \frac{m}{s}$

$c \sim 3E8\ \frac{m}{s}$

speed of light measurement

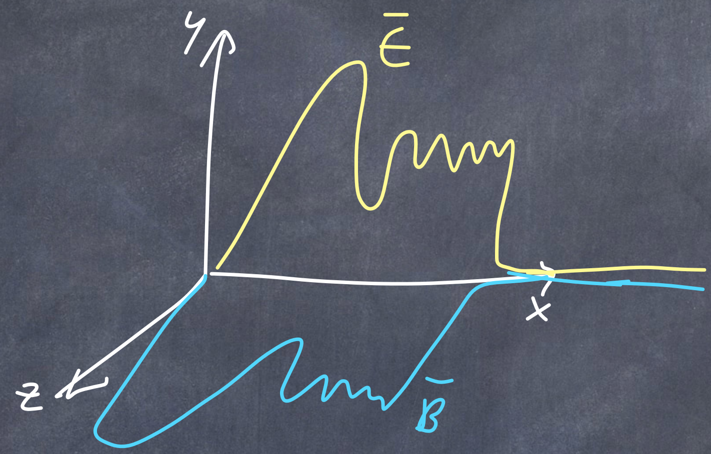


Δx = path length difference = $2L_2 - 2L_1$

Δt = time difference recorded on oscilloscope

$$c = \frac{\Delta x}{\Delta t} = \sim 3\text{e}8 \ \frac{m}{s}$$

The shape does not need to be sinusoidal. It can be a weird pulse:



But we can represent any wave or pulse by superpositions of sine waves (using* Fourier transforms) so it is convenient to consider EM waves as sine waves.

* not covered in this class, but in script.

Note: wave equations

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0}$$ (3)

and

$$\frac{\partial^2 B_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2} = 0$$

The wave functions (e.g., eq (0)) satisfy wave equations for all x and t.

Solving Maxwell's equations, we find that the velocity of the waves is related to ε_0 & μ_0 (in vacuum)

(4) $$\boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$ in a vacuum.

This is pretty cool! If we measure the electric field of a charge, we can determine ε_0. If we measure the magnetic field of an electric current, we can determine μ_0. Together, these "predict" the speed of light.

In a medium, with ε, μ ($\varepsilon = k\varepsilon_0$, $\mu = k_\mu \mu_0$)

$$c' = \frac{1}{\sqrt{\varepsilon \mu}}$$

The ratio of the speed of light in a medium compared to that in a vacuum is equal to "n"

(5) $$\boxed{\frac{c}{c'} = \frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon_0 \mu_0}} = n}$$

$n > 1$ (for air, $n = 1.00029$)

"n": index of refraction

All EM waves obey $f\lambda = c$

There are different wavelengths of light

visible light: λ: 400 nm $\longleftrightarrow$ 700 nm

400 E-9m

blue red

$$f = \frac{c}{\lambda} = \frac{3E8\ \frac{m}{s}}{600E-9m} \sim 5E14\ Hz = 500\ THz$$

Table 13.1: Rough classification of electromagnetic spectrum and its applications.

Class	Wavelength λ	Frequency f	Application
Radio waves	$> 1\,\mathrm{m}$	$< 300\,\mathrm{MHz}$	Radio & TV broadcast, telecomunications maritime navigation
Microwaves	$1\,\mathrm{mm} - 1\,\mathrm{m}$	$300\,\mathrm{MHz} - 300\,\mathrm{GHz}$	Microwave oven, radar, mobile phones, 4G, Wi-Fi, satellite communications, GPS, cosmic microwave background
Infrared	$750\,\mathrm{nm} - 1\,\mathrm{mm}$	$300\,\mathrm{GHz} - 400\,\mathrm{THz}$	Thermal imaging, TV remote control, night vision, bio imaging, optical fibers
Visible	$400\,\mathrm{nm} - 750\,\mathrm{nm}$	$400\,\mathrm{THz} - 750\,\mathrm{THz}$	Human vision, illumination, photography, microscopes, lasers
Ultraviolet	$10\,\mathrm{nm} - 400\,\mathrm{nm}$	$750\,\mathrm{THz} - 30\,\mathrm{PHz}$	Disinfection, dental curing, black lights, sun tanning, counterfeit detector
X rays	$0.01\,\mathrm{nm} - 10\,\mathrm{nm}$	$30\,\mathrm{PHz} - 30\,\mathrm{EHz}$	Crystallography, radiation therapy, medical imaging, security scans
Gamma rays	$< 0.01\,\mathrm{nm}$	$> 30\,\mathrm{EHz}$	Radioactive sources, cancer treatments, PET scans, cargo container screening

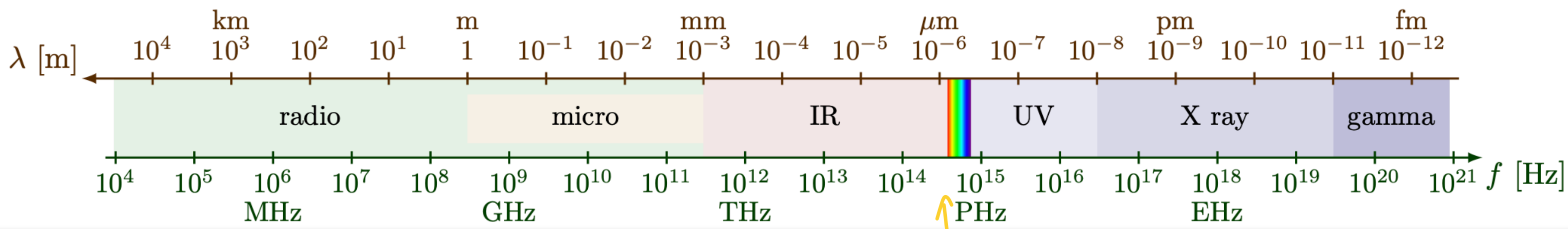
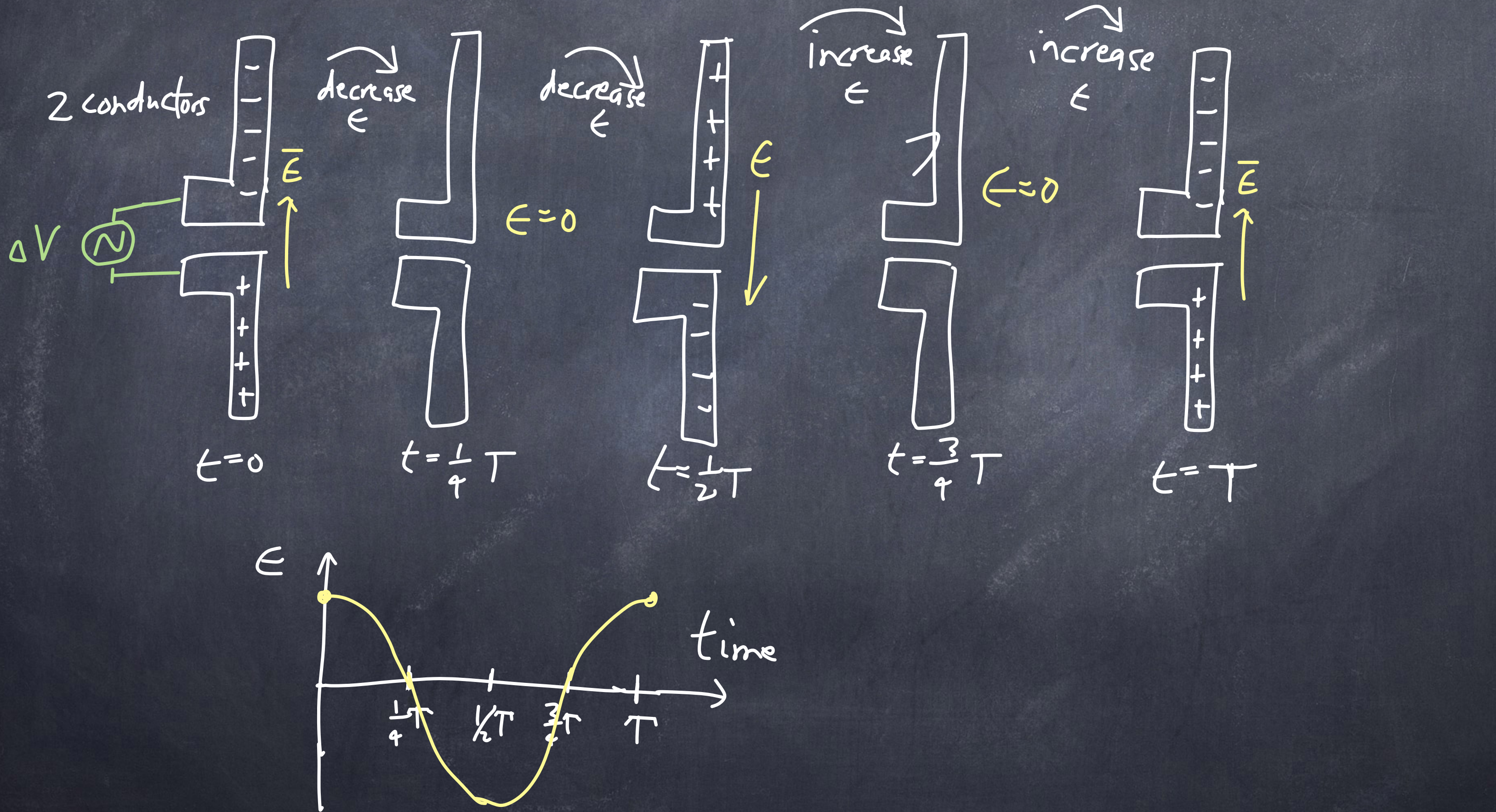
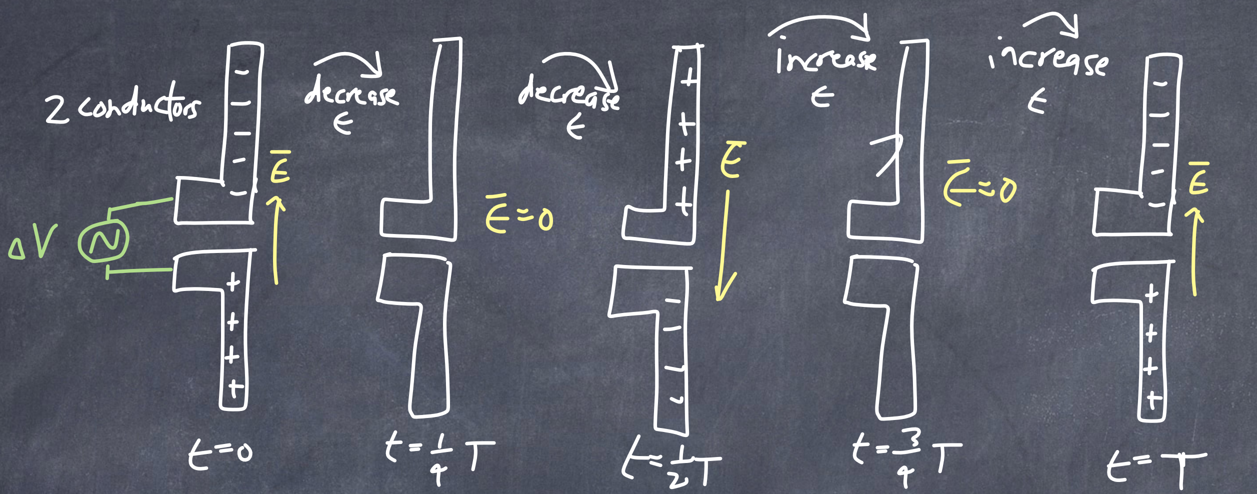


Figure 13.2: Electromagnetic spectrum with rough classification.

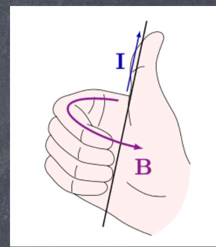
How to create light from moving an electric charge.

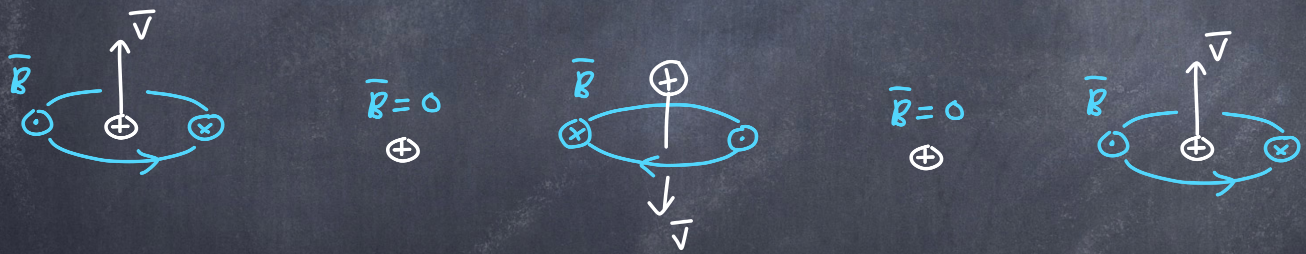
To make an EM wave, we can accelerate an electric charge.





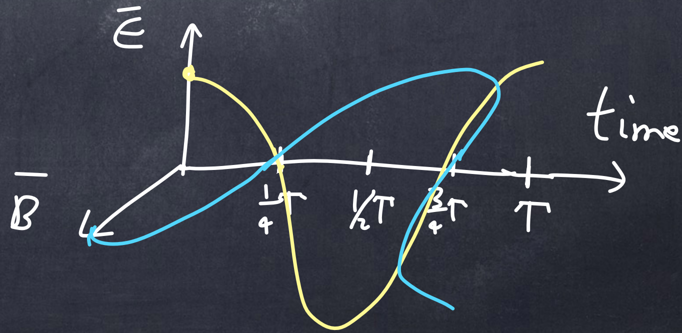
Moving electric charges generate magnetic fields.





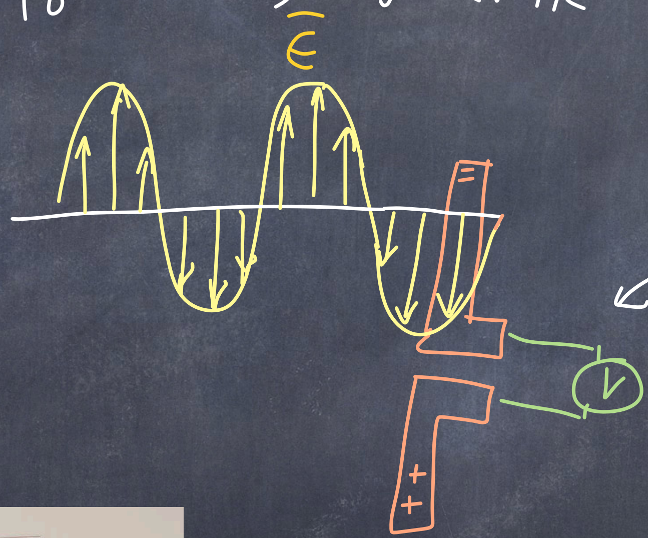
If you make a $\bar{B} \perp$ to an $\bar{E}$ field, you get light!

$\bar{E} \perp \bar{B}$

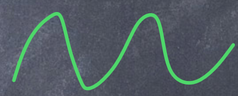


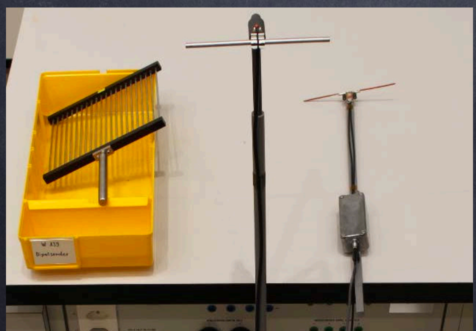
We have made an EM wave. Detecting an EM wave is the opposite of making one.

The $\bar{E}$ Field of the wave causes electric charges to move, generate an alternating current



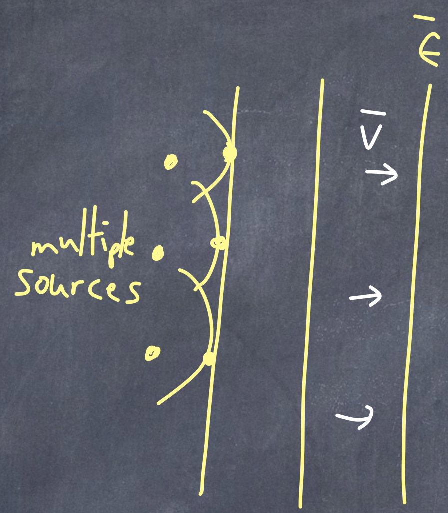
measure

we see 



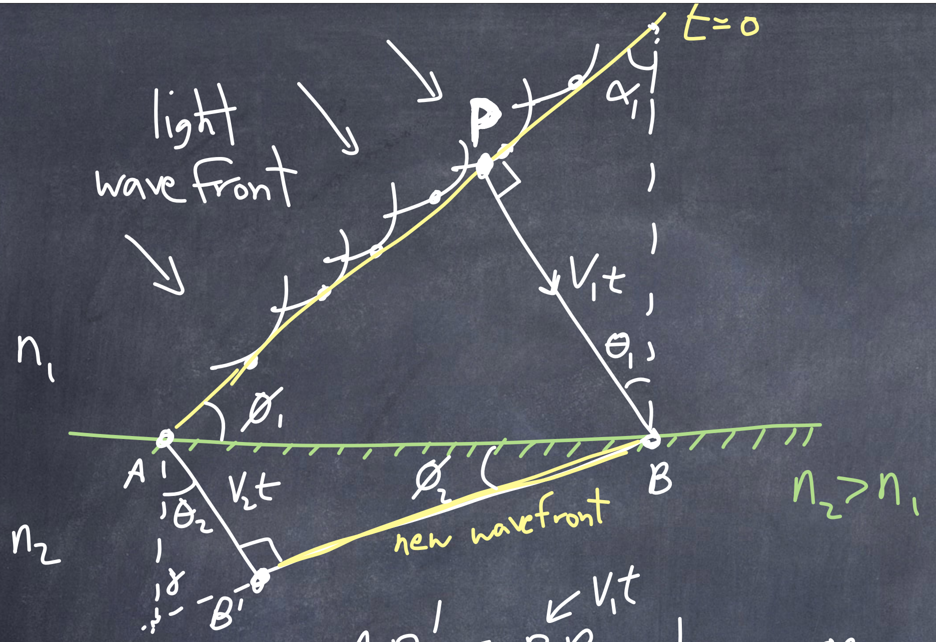
Light propagates in spherical way according to Huygen's principle

plane waves:



Consider a plane wave entering a medium with higher n. Remember ⑤

velocity of EM wave in the medium is $c' = \frac{c}{n}$



$V_1 = \frac{c}{n_1} \qquad V_2 = \frac{c}{n_2}$

In time time t, the wave propagates a distance.

$$\left.\begin{aligned} \phi_1 + \alpha_1 + 90^\circ &= 180^\circ \\ \theta_1 + \alpha_1 + 90^\circ &= 180^\circ \end{aligned}\right\} \rightarrow \phi_1 = \theta_1$$

$$\left.\begin{aligned} \phi_2 + \gamma + 90^\circ &= 180^\circ \\ \theta_2 + \gamma + 90^\circ &= 180^\circ \end{aligned}\right\} \rightarrow \phi_2 = \theta_2$$

$AB' < PB$ because the velocity is slower in n_2 than n_1

$V_2 t$

The new wavefront has changed direction!

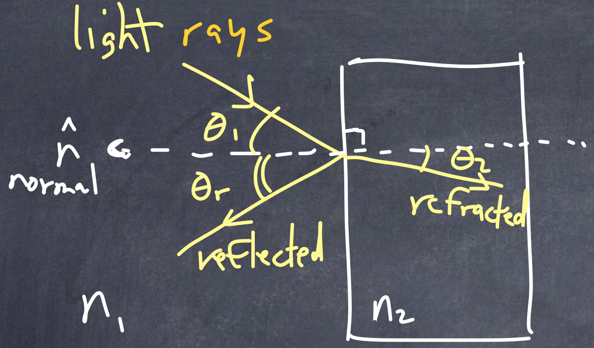
$$\left.\begin{aligned} &\text{Note } \sin\phi_1 = \frac{V_1 t}{AB} \\ &\text{But also } \sin\phi_2 = \frac{V_2 t}{AB} \end{aligned}\right\} \quad AB = \frac{V_1 \cancel{t}}{\sin\phi_1} = \frac{V_2 \cancel{t}}{\sin\phi_2} \qquad \begin{aligned} \phi_1 &= \theta_1 \\ \phi_2 &= \theta_2 \end{aligned}$$

$V_1 = \frac{c}{n_1} \qquad V_2 = \frac{c}{n_2}$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Snell's Law

The law of refraction



Light reflects and refracts

reflection : $\theta_1 = \theta_r$

refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

where n_1 + n_2 are the indices of refraction such that $n = \frac{c}{v}$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad v = \frac{1}{\sqrt{\varepsilon \mu}}$$

If $\theta_1 = 0°$, $\perp$ to the surface,

the reflected intensity, $I_r = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 I_0$, then the rest is transmitted (refracted)

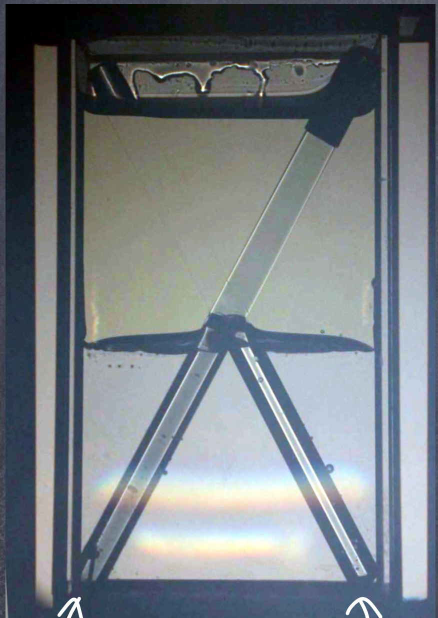
As an example for air ($n \sim 1$), glass ($n \sim 1.5$)

then $\frac{I_r}{I_0} = 4\%$

The smaller the $n_1 - n_2$, the difference, the less is reflected.

If $n_1 = n_2$, no reflected light.

$n_1 = n_2$ for one of the glasses

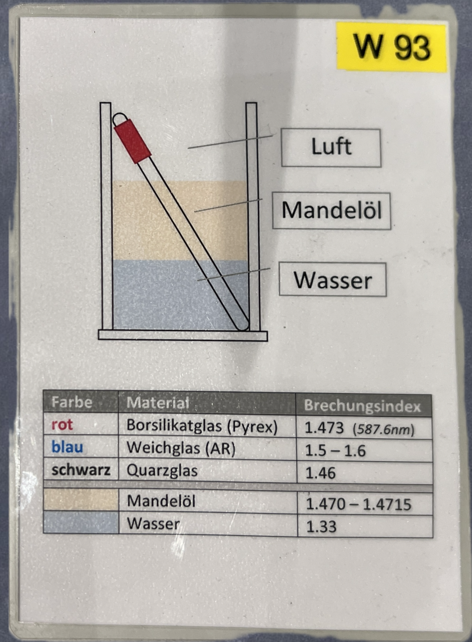


oil

water

glass one

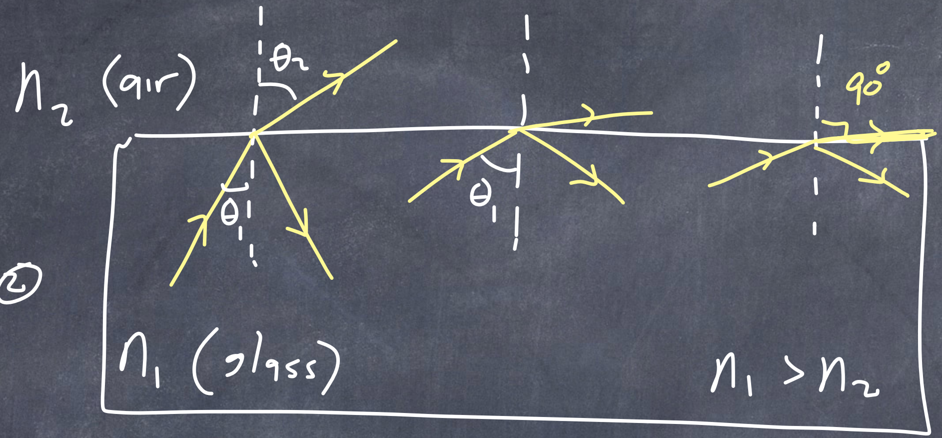
glass two

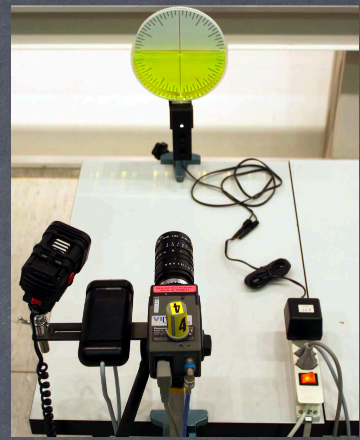


Farbe	Material	Brechungsindex
rot	Borsilikatglas (Pyrex)	1.473 (587.6nm)
blau	Weichglas (AR)	1.5 – 1.6
schwarz	Quarzglas	1.46
	Mandelöl	1.470 – 1.4715
	Wasser	1.33

Reflection and refraction of light

light goes from ① → ②
n_1 to n_2





As θ_1 increases for $n_1 > n_2$, θ_2 increases more.
When $\theta_2 = 90^\circ$, no light is transmitted.

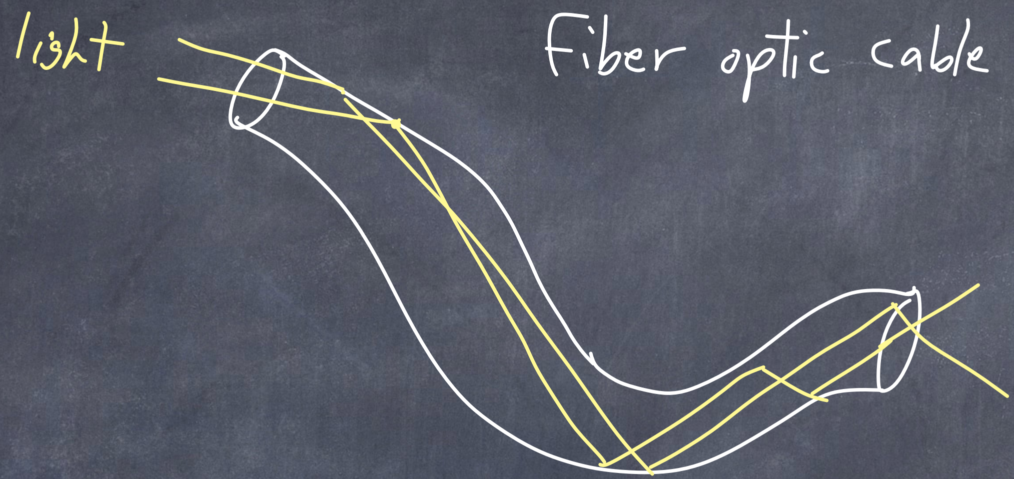
$$n_1 \sin\theta_1 = n_2 \underbrace{\sin\theta_2}_{\sin 90^\circ = 1}$$

$$\boxed{\text{critical angle} \quad \sin\theta_c = \frac{n_2}{n_1} \quad \text{for } n_1 > n_2}$$

θ_c is the critical angle, or θ_1 where above θ_c all light is reflected.

Since $\sin\theta$ cannot be >1,
this value is undefined for $n_2 > n_1$
(does not happen)

Above θ_c, we have total internal reflection



fiber $n \sim 1.4$

$\rightarrow \theta_c \sim 45^\circ$

In refraction, the speed of light changes, $V = \frac{c}{n}$

But the frequency of the light stays the same.

This is because the atoms that absorb the light and re-transmit the light, vibrate and emit light at the $\underbrace{\text{same frequency}}_{\text{to do with atomic energy levels ...}}$

we have n_1, n_2

$$f_1 = f_2$$

$$V_1 = \frac{c}{n_1} \qquad V_2 = \frac{c}{n}$$

Since $f\lambda = V$, the wavelength must change in a material with different n.

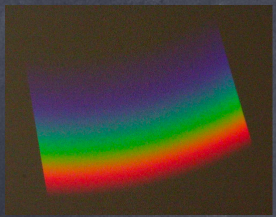
$$f_1 = \frac{V_1}{\lambda_1} = f_2 = \frac{V_2}{\lambda_2} \Rightarrow \frac{\cancel{c}}{n_1 \lambda_1} = \frac{\cancel{c}}{n_2 \lambda_2} \Rightarrow \left[\lambda_2 = \lambda_1 \frac{n_1}{n_2}\right]$$

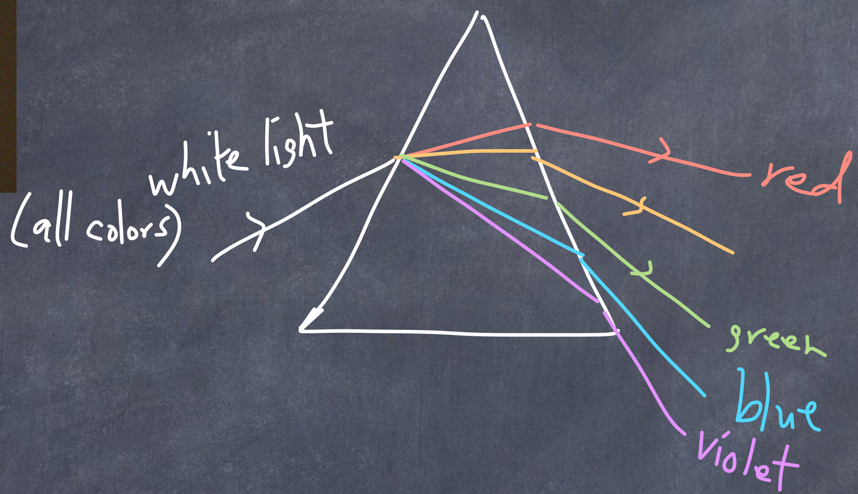
change in wavelength inside material 2

For air (n_1) to water (n_2) wavelengths get smaller.

Light gets "more blue" under water.

In addition) the index of refraction depends slightly on the wavelength of the light.





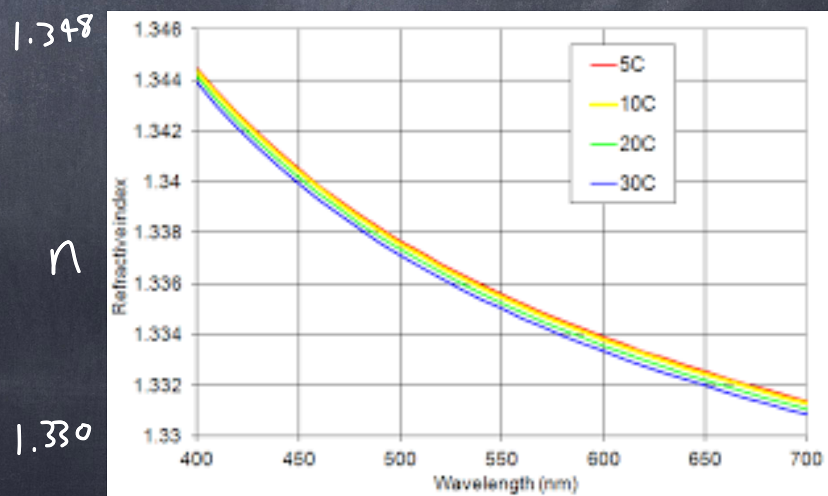
(exaggerated)

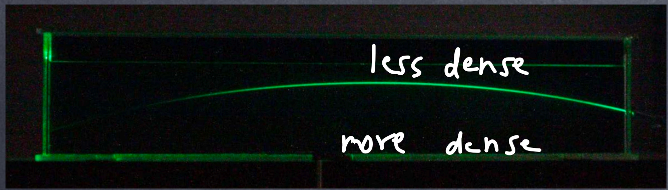
short wavelength is bent more

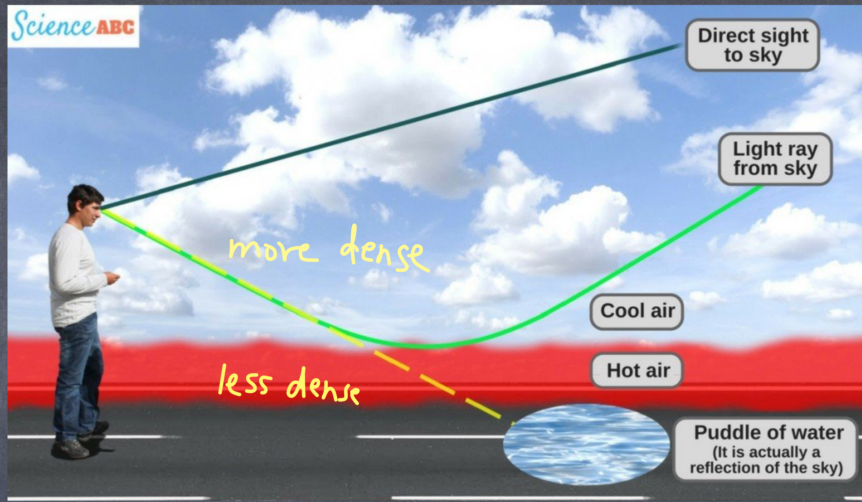
blue bent more than red.

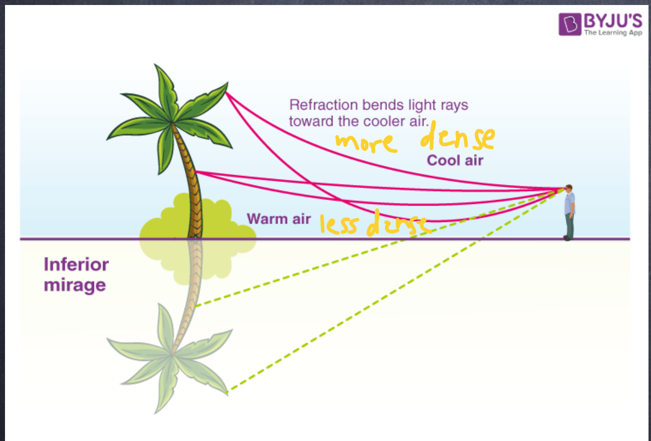
Also, n depends on the density.

Hotter air is less dense.

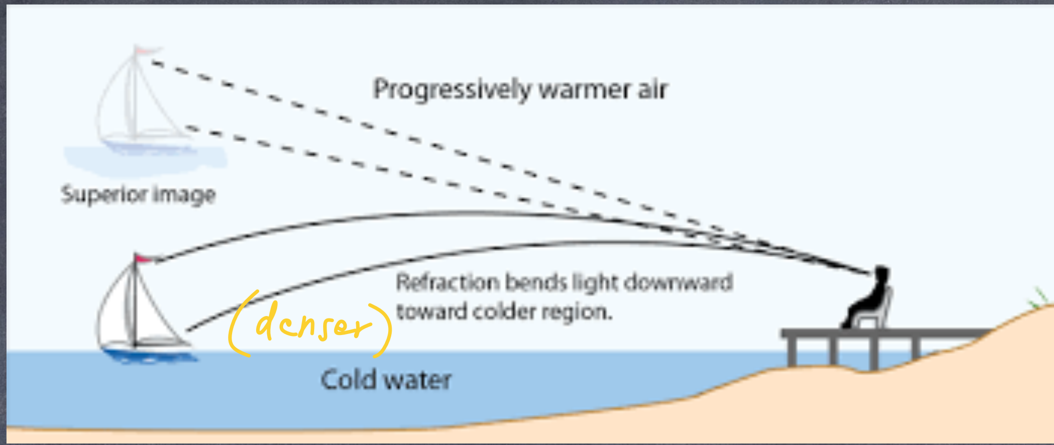



less dense
more dense


Science ABC
Direct sight to sky
Light ray from sky
more dense
Cool air
Hot air
less dense
Puddle of water
(It is actually a reflection of the sky)


BYJU'S
Refraction bends light rays toward the cooler air.
more dense
Cool air
Warm air
less dense
Inferior mirage


more dense
less dense



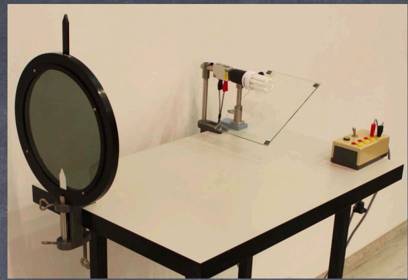
less dense →

more dense →

David Morris took the photographs from the hamlet of Gillan, near Falmouth

"looming"


W51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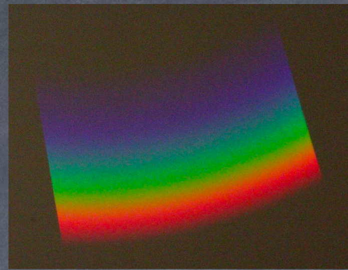
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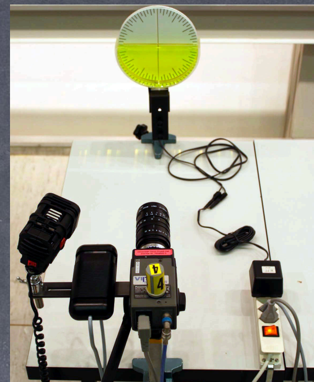
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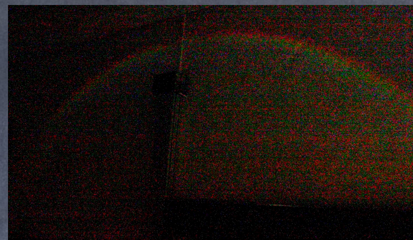
W77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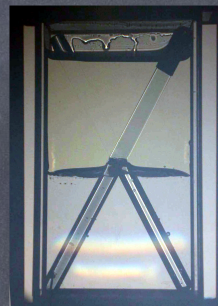
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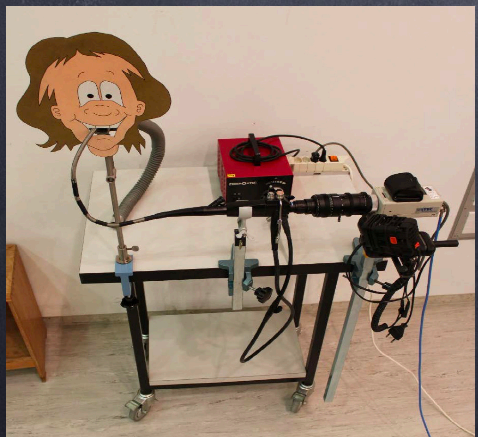
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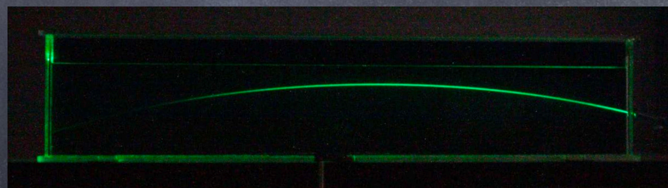
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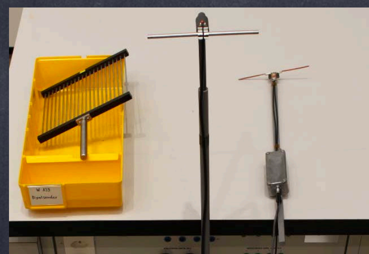
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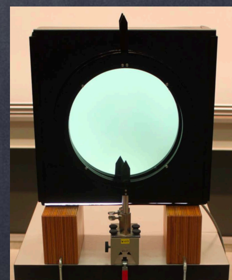
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